

# DILUTING GRAVITY WITH COMPACT HYPERBOLOIDS

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I give a brief informal introduction to the idea and tests of large extra dimensions, focusing on the case in which the space-time manifold has a direct product structure. I then describe some attractive implementations in which the internal space comprises a compact hyperbolic manifold. This construction yields an exponential hierarchy between the usual Planck scale and the true fundamental scale of physics by tuning only  $\mathcal{O}(1)$  coefficients, since the linear size of the internal space remains small. In addition, this allows an early universe cosmology with normal evolution up to substantial temperatures, and completely evades astrophysical constraints.

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## 1 Some Background about Large Extra Dimensions

There are a number of talks in this conference about the idea of large extra dimensions. With this in mind I will begin with a brief overview of the general idea and of the constraints on it.

### 1.1 Motivations : the Hierarchy Problem

The hierarchy problem, for our purposes, is usually posed in the following way: Why is gravity so much weaker than the other forces? To make this concrete compare

$$G_N = M_{\text{pl}}^{-2} \sim 10^{-33} \text{ GeV}^{-2} \quad (1)$$

with

$$G_F = M_W^{-2} \sim 10^{-5} \text{ GeV}^{-2} \quad (2)$$

Phrased in this way, the Planck mass  $M_{\text{pl}}$  is considered to be the fundamental scale of physics and the puzzle is the comparative smallness of  $M_W$ , or the comparative strength of the electroweak force. The real problem here is not so much the fact that the two scales differ so drastically, but rather that such an arrangement is doomed to be ruined by radiative corrections in a generic quantum field theory. Thus, theoretical efforts have mostly focused on restoring the *technical* naturalness of this vast difference in scales via, for example, supersymmetry, in which the offending quadratic divergences are absent.

Given this suggestive phrasing, a logical alternative statement of the problem becomes clear. If we instead imagine that the fundamental scale of physics  $M_*$  is close to the weak scale  $M_W$ , then the hierarchy problem becomes: Why is  $M_{\text{pl}}$  so much larger? When considering this possibility however, one must keep in mind some important facts about physics at these scales:

- The Electroweak interactions have been tested up to energies  $E \sim M_W$ , or equivalently, down to scales  $d \sim M_W^{-1} = 10^{-16}\text{cm}$
- Gravity, in the form of Newton's law has been tested down to a scale  $d \sim 10^{33}/M_{\text{pl}} \sim 1\text{cm}$

Thus, we know that there are no quantum gravity effects up to energies  $E \sim M_W$ , since, for example, there is no evidence of energy lost to gravitons in  $e^+e^-$  annihilations.

Given these constraints, let us examine how it might be that the Planck mass is a comparatively huge derived quantity in a theory in which the weak scale is fundamental. The fundamental idea behind the new constructions is the following<sup>1,2</sup>. Imagine that space time has  $3 + 1 + d$  dimensions, and that  $d$  of these are compact while the remaining  $3 + 1$  comprise the familiar space-time in which we appear to live. Now make the following two crucial assumptions:

- Standard model particles are *confined* to the  $3 + 1$  dimensional sub-manifold.
- Gravity is not confined, and therefore gravitons propagate in the bulk

These assumptions will turn out to be crucial, since to obtain the necessary hierarchy we will require large volume extra dimensions, and these are ruled out by the precision tests mentioned above if the weak interactions can take place in the bulk. While such a structure may seem somewhat unnatural from the phenomenological viewpoint taken here, it is important to mention that the appropriate behavior occurs readily in some compactifications of string theory. In particular, in Hořava-Witten theory<sup>3</sup>, precisely this division of interactions occurs. The essential M-theoretic ingredient responsible for this is the existence of D-branes, non-perturbative extended objects, in the string spectrum<sup>4</sup>. Open strings must end on D-branes (with “D”irichlet boundary conditions on their endpoints), while the open strings are free to propagate through the whole larger space. Since the excitations of open strings correspond to standard model-like degrees of freedom, while the closed string

excitations describe the geometric (gravitational) degrees of freedom, the resulting structure is one with standard model fields confined to a D-brane, and gravity propagating in the full space.

To be concrete, let us describe the original large extra dimension scenario. Imagine that the full space-time manifold is

$$\mathcal{M}^{3+1} \times T^d, \quad (3)$$

where  $\mathcal{M}^{3+1}$  describes a flat Minkowski or cosmological space which we observe, and  $T^d$  is a  $d$ -dimensional torus of common linear size  $R$  describing the internal space. Consider how gravitational flux spreads out around a point mass  $m$ , as in figure 1.

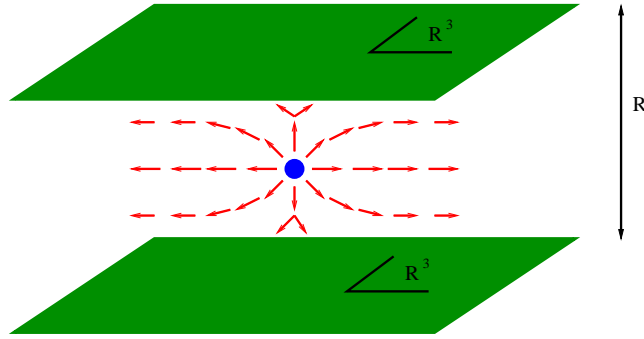


Figure 1. Gravitational flux around a point mass in a direct product space with one extra dimension.

The gravitational acceleration experienced by a test particle at distance  $r$  from such a point mass obeys the following

$$\begin{aligned} |\mathbf{g}| &= \frac{G_{(4+d)}m}{r^{2+d}} \quad \text{for } r \ll R \\ &= \frac{G_{(4+d)}m}{R^d r^2} \quad \text{for } r \gg R. \end{aligned} \quad (4)$$

This is easy to understand in terms of Gauss' law : On small scales  $r \ll R$  gravity is diluted by spreading out into all the dimensions, whereas on large scales  $r \gg R$  gravity is smeared out over the internal space and can only spread into the  $3 + 1$  dimensional space.

Now, since we understand gravity extremely well on large scales, we must identify

$$G_4 \equiv \frac{G_{(4+d)}}{R^d} \quad \Leftrightarrow \quad M_{\text{pl}}^2 \equiv M_*^2 R^d. \quad (5)$$

Thus, the observed Planck mass can be huge, even if the fundamental scale of physics is of order a TeV. All that is needed is that the *volume* of the extra dimensional space (here  $R^d$ ) is large enough. In particular, for  $M_* \sim 1$  TeV, we require

$$R \sim 10^{\frac{32}{d}-17} \text{cm} . \quad (6)$$

For example,

- If  $d = 1$ , we need  $R \sim 10^{13}$  cm, which is obviously excluded by any number of large scale tests of gravity.
- If  $d = 2$ , we need  $R \sim \text{mm}$ , a truly large extra dimension.

So, in a picture with extra dimensions and standard model particles restricted to the brane, the hierarchy problem can be recast in the interesting guise of a mismatch of spatial scales, rather than one of energies.

### 1.2 Constraints

The constraints on such a possible structure for space-time and particle physics come from three main sources: the laboratory, astrophysics and cosmology. In the laboratory<sup>a</sup>, the relevant quantity is the amplitude for single graviton emission:

$$\mathcal{A} \sim \sqrt{M_*^{-(d+2)}} . \quad (7)$$

This allows one to write the dimensionless graviton emission rate for a process of energy  $\Delta E$  as

$$\mathcal{R} \sim G_{(4+d)} \Delta E^{d+2} \equiv \left( \frac{\Delta E}{M_*} \right)^{d+2} . \quad (8)$$

There are two important facts that we shall need from this expression. One is that the constraint becomes weaker the more extra dimensions there are, and the second is that the constraint is stronger for higher values of  $\Delta E$ . As it turns out, collider constraints<sup>6</sup> are currently not the strongest ones faced by the large extra dimension picture. This can be seen, for example, by considering the process  $K \rightarrow \pi + \text{graviton}$ , for which the branching ratio is

$$\text{Br}(K \rightarrow \pi g) \sim \left( \frac{m_K}{M_*} \right)^{d+2} \sim 10^{-12} \quad \text{for } d = 2 . \quad (9)$$

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<sup>a</sup>For a nice summary of laboratory constraints see <sup>5</sup>

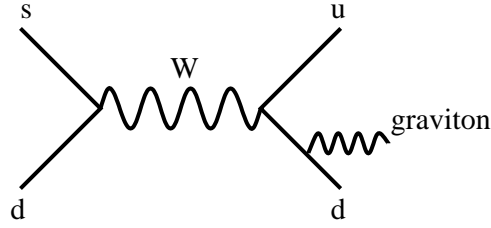


Figure 2. A feynman diagram for  $K \rightarrow \pi + \text{graviton}$ .

More important are constraints arising from astrophysics. The basic issue is that the Kaluza-Klein (KK) modes in the model are light  $M_{KK} \geq R^{-1} \geq 10^{-4}\text{eV}$ , and numerous  $N_{KK} \simeq M_{\text{pl}}^2/M_*^2 \leq 10^{32}$ . Thus, although each KK mode is very weakly coupled, of order  $1/M_{\text{pl}}$ , to standard model particles on the brane, there are so many of them that they can be copiously produced by energetic processes on the brane. Therefore, it is possible for astrophysical bodies to lose energy by emitting gravitons into the extra dimensions. The most important ways in which this can occur have been termed *gravistrahlung*, referring to the production of a KK graviton in heavy nucleon collisions, and the *gravi-Primakoff* process, in which a KK graviton is produced through a photon scattering from a heavy nucleus (see figure 3). As mentioned earlier,

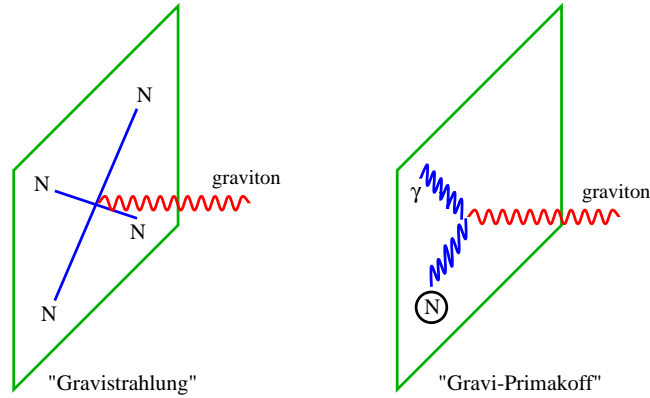


Figure 3. Two processes through which astrophysical bodies can lose energy to KK gravitons.

the most energetic processes yield the tightest constraints, and so it is useful

to consider energy loss from the most energetic astrophysical event yet known, supernova 1987A (SN1987A). A careful analysis<sup>7</sup> yields

$$M_* \geq 50 \text{ TeV} \quad \text{for } d = 2 \quad (10)$$

(and less important constraints on higher values of  $d$ ). As is often the case in modern particle physics, some of the most important constraints on this structure arise from cosmological considerations. I will just mention two main issues here. As I have described, extra dimensions provide an alternative mechanism through which astrophysical bodies can cool. This remains true for the universe itself. In the standard cosmology, the universe cools adiabatically as the scale factor  $a(t)$  increases, with the temperature-time relationship (in the radiation-dominated era)

$$\frac{\dot{T}}{T} \sim \frac{\dot{a}}{a} = H \sim \frac{T^2}{M_*} . \quad (11)$$

In the new picture, there is a competing evaporative cooling mechanism due to *cosmic gravistrahlung*, with temperature-time relationship

$$\frac{\dot{T}}{T} \sim \frac{T^{d+3}}{M_*^{d+2}} . \quad (12)$$

Since the standard cosmology is so successful as a description of the universe at temperatures below that of, for example, primordial nucleosynthesis, we require that the new cooling mechanism be sub-dominant at least at temperatures below that. To this end it has become customary to define the *normalcy temperature*  $T_*$ , to be that temperature below which evaporative cooling is negligible compared to the usual adiabatic mechanism. In the basic large extra dimension scenario that I am reviewing here, this temperature is

$$\begin{aligned} T_* &\sim 10 \text{ MeV} \quad \text{for } d = 2 \\ &\sim 10 \text{ GeV} \quad \text{for } d = 6 \text{ (string theory)} . \end{aligned} \quad (13)$$

The second cosmological constraint arises from the possibility of gravitino overproduction. If the underlying theory is supersymmetric, one expects gravitinos to be thermally produced. If they are produced at temperature  $T$ , they have a lifetime

$$t \sim \frac{M_{\text{pl}}^2}{T^3} . \quad (14)$$

If  $M_*$  is too small, these particles may over-close the universe or distort the  $\gamma$ -ray background. These constraints are most relevant for  $d = 2$ , in which

case they yield

$$\begin{aligned} M_* &> \mathcal{O}(\text{TeV}) \\ &> 110 \text{ TeV} . \end{aligned} \tag{15}$$

To summarize the combined constraints from the laboratory, astrophysics and cosmology;  $d = 1$  is experimentally ruled out (easily),  $d = 2$  is quite constrained, and  $d > 2$  is relatively unconstrained.

## 2 Some Interesting Manifolds

I have spent some time giving an overview of the basic concepts behind and constraints on the large extra dimension model, as originally proposed. I would now like to shift gears a little, and describe how these ideas might be extended to the case when the internal manifold is no longer flat (merely a torus)<sup>8</sup>. More specifically I will be interested in topologically nontrivial internal spaces, and in particular in the case that the internal space is a compact hyperbolic manifold.

Compact hyperbolic manifolds (CHMs)<sup>9</sup>, are obtained from  $H^d$ , the universal covering space of hyperbolic geometry (that admitting constant negative curvature), by modding out by an appropriate freely acting discrete subgroup  $\Gamma$  of the isometry group of  $H^d$ . (If  $\Gamma$  is not freely-acting, then the resulting quotient is a non-flat non-smooth orbifold. Such a structure may be related to the Randall-Sundrum models<sup>10</sup>)

Consider space-times of the form

$$\mathcal{M}^4 \times (H^d/\Gamma|_{\text{free}}) , \tag{16}$$

with  $\mathcal{M}^4$  a Friedmann, Robertson-Walker (FRW) 4-manifold, with metric

$$G_{IJ}dz^I dz^J = g_{\mu\nu}^{(4)}(x)dx^\mu dx^\nu + R_c^2 g_{ij}^{(d)}(y)dy^i dy^j . \tag{17}$$

In this expression  $R_c$  is the physical curvature radius of the CHM, so that  $g_{ij}(y)$  is the metric on the CHM normalized so that its Ricci scalar is  $\mathcal{R} = -1$ , and  $\mu = 0, \dots, 3$ ,  $i = 1, \dots, d$ .

Locally negatively curved spaces exist only for  $d \geq 2$ , and the properties of CHMs are well understood only for  $d \leq 3$ . However, it is known that CHMs in dimensions  $d \geq 3$  possess the important property of *rigidity*<sup>11</sup>. As a result, these manifolds have *no massless shape moduli*, implying that the volume of the manifold, in units of the curvature radius  $R_c$ , cannot be changed while maintaining the homogeneity of the geometry. Therefore, the stabilization of such internal spaces reduces to the problem of stabilizing a single modulus, the curvature length or the “radion”.

Although CHMs may seem like quite abstract objects, their popularity among mathematicians has led to some useful tools for visualizing their structure. In figure 4 I have used the Geomview package<sup>12</sup> to display two examples of CHMs generated by Jeff Weeks' SnapPea program<sup>13</sup>. These are presented

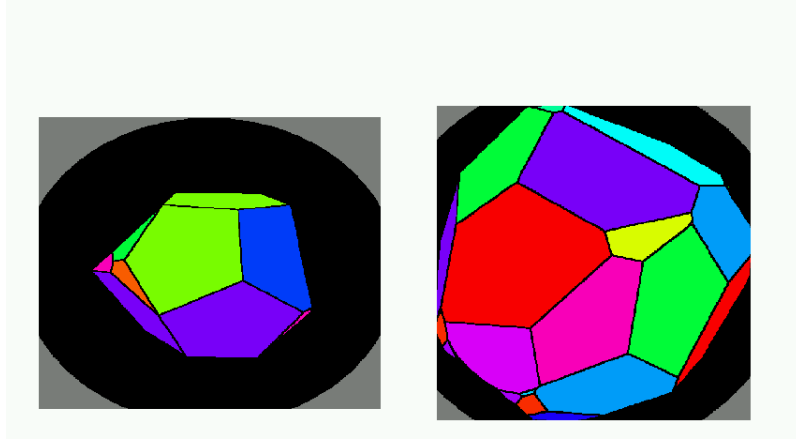


Figure 4. Two CHMs, one small volume, the other large volume.

in the Poincaré metric, and what is important here is that those parts of the manifold that are identified under actions under  $\Gamma$  are shaded in the same way. I would just like to draw attention to two important features. First, the second example has a much larger volume than the first in units of the curvature radius. In addition, the second example is considerably more topologically complex than the first, as can be seen by the much higher number of identifications under  $\Gamma$ . These features are interrelated, and are quite general.

### 2.1 Compact Hyperbolic Spaces and Volume

The central feature of CHMs that I will exploit here is the behavior of their volume as a function of linear size in the manifold. A specific example that it is useful to keep in mind is a 3-sphere of radius  $r$ , cut out of an  $H^3$  of curvature radius  $R_c$ . The volume of such a sphere can be calculated exactly, and is given by

$$\text{Vol}(r) = \pi R_c^3 \left[ \sinh \left( \frac{2r}{R_c} \right) - \frac{2r}{R_c} \right]. \quad (18)$$

It is useful to examine this expression in two limits. When  $r \ll R_c$ , the first term in a Taylor expansion yields

$$\text{Vol}(r) \sim \frac{4}{3}\pi r^3, \quad (19)$$

as one would expect, since the manifold looks flat on these scales. However, in the opposite limit  $r \gg R_c$ , we obtain

$$\text{Vol}(r) \sim \frac{\pi}{2} R_c^3 e^{2r/R_c}, \quad (20)$$

and therefore the volume of the sphere grows exponentially with linear size for large radius.

This result remains true for other compact spaces constructed from  $H^3$ . In general, the total volume of a smooth compact hyperbolic space in any number of dimensions is

$$\text{Vol}(\text{CHM}) = R_c^d e^\alpha, \quad (21)$$

where  $\alpha$  is a constant, determined by topology. Since the topological invariant  $e^\alpha$  characterizes the volume of the CHM, it is also a measure of the largest distance  $L$  around the manifold. Although CHMs are globally anisotropic, since the largest linear dimension gives the most significant contribution to the volume, one can employ eq. (18), or its generalizations to  $d \neq 3$ , to find an approximate relationship between  $L$  and  $\text{Vol}(\text{CHM})$ . For  $L \gg R_c/2$  the appropriate asymptotic relation, dropping irrelevant angular factors, is

$$e^\alpha \simeq \exp \left[ \frac{(d-1)L}{R_c} \right]. \quad (22)$$

In a model with such a manifold as an extra dimensional space, this relationship allows us to compute the expression for  $M_{\text{pl}}$  in terms of linear distance in the space, to compare with (5) in the flat case. The relevant expression is

$$M_{\text{pl}}^2 = M_*^{2+d} R_c^d e^\alpha \simeq M_*^{2+d} R_c^d \exp \left[ \frac{(d-1)L}{R_c} \right]. \quad (23)$$

Thus, in strong contrast to the flat case, in which  $M_{\text{pl}}$  has a power law dependence on linear size, with a CHM the relationship is exponential. As I'll mention later, the most reasonable and interesting case is the smallest possible curvature radius,  $R_c \sim M_*^{-1}$ , since this is the only scale available in the problem. Taking  $M_* \sim \text{TeV}$  then yields

$$L \simeq 35 M_*^{-1} = 10^{-15} \text{mm}. \quad (24)$$

Therefore, one of the most attractive features of a CHM internal space is that *to generate an exponential hierarchy between  $M_* \sim \text{TeV}$ , and  $M_{\text{pl}}$  requires only that the linear size  $L$  be very mildly tuned.*

## 2.2 Eigenmodes and Kaluza-Klein Excitations

I have tried to convince you that CHMs provide an attractive alternative manifold for implementing large extra dimension ideas. However, if this idea is to be taken seriously it is necessary to examine the constraints and possible experimental signatures.

To uncover the relevant physics of these models one must consider the spectrum of small fluctuations  $h$  in the metric around the background metric. Writing

$$G_{IJ} \rightarrow G_{IJ} + e^{ip \cdot x} h_{IJ}(y). \quad (25)$$

one sees 3 different types of KK fluctuations

- $h_{\mu\nu}$ , the spin-2 piece;
- $h_{ij}$ , with indices only in the internal directions, giving spin-0 fields for the 4D observer;
- $h_{i\mu}$ , the mixed case, giving spin-1 4D fields.

The 4D KK masses of these states are the eigenvalues of the appropriate internal-space Laplacians acting on  $h(y)$ . For the spin-2 case the relevant operator is the Laplace-Beltrami operator  $\Delta_{LB}$  (the Laplacian on scalar functions in the internal space), defined by

$$\Delta_{LB}\phi(y) = |g(y)|^{-1/2} \partial_i \left( |g(y)|^{1/2} g^{ij} \partial_j \phi(y) \right). \quad (26)$$

Although there are no known analytic expressions for the individual eigenvalues of  $\Delta_{LB}$  on a CHM of any dimension, some generic properties are known. First, a variational argument shows that the spectrum of  $\Delta_{LB}$  is bounded from below, and the lowest eigenmode is just the constant function on the CHM. This zero mode is the internal space wave-function of the massless spin-2 4D graviton.

Second, since the internal space is compact, the spectrum of  $\Delta_{LB}$  on a CHM is discrete and has a gap between the zero mode and the first excited KK state. The size of this gap is all important. A crucial point is that most of the eigenmodes of  $\Delta_{LB}$  on a CHM have wavelengths less than  $R_c$ , and the number density of these modes is well approximated by the usual Weyl asymptotic formula<sup>b</sup>

$$n(k) = (2\pi)^{-d} \Omega_{(d-1)} V_d k^{d-1}, \quad (27)$$

<sup>b</sup>There can also be a few lighter *supercurvature modes*, with wavelengths as large as the longest linear distance in the manifold, and masses thus bounded below by  $L^{-1}$ .

where  $\Omega_{(d-1)} = \text{Area}(S^{d-1})$ . Further, bounds on the value of the first non-zero eigenvalue are known. In the best-studied CHM case of  $d = 2$  it can be proven that a large enough volume (and thus genus)  $d = 2$  CHM will have first eigenvalue  $\geq 171/(784R_c^2)$ . The analogous conjecture in  $d = 3$  is more problematic, but has also been made<sup>14</sup>. In addition, numerical studies of the spectra of even small volume  $d = 3$  CHMs show that they have very few modes with  $\lambda < R_c$ <sup>15</sup>.

The basic result is that the first KK modes are exponentially more massive than those in the flat case. This drastically raises the threshold for their production, and as a consequence the astrophysical bounds<sup>16,7</sup> completely disappear since the lightest KK mode has a mass (at least 30 GeV), much greater than the temperature of even the hottest astrophysical object. Similarly the large KK masses imply a much higher normalcy temperature  $T_*$ , up to which the evolution of our brane-localized 4D universe can be normal radiation-dominated FRW.

Turning to the spin-0(1) excitations, the detailed form of the Laplacian is modified. However, the Mostow-Prasad rigidity theorem for CHMs of dimension  $d \geq 3$  implies that  $\Delta_{\text{LL}}$  has no zero modes, and it is conjectured that the gap to the first excited state is of similar size to the Laplace-Beltrami case, a result that is physically reasonable. Finally for the spin-1 fluctuations  $h_{i\mu}$  recall that any zero modes would correspond to KK gauge-bosons and are directly related to the continuous isometries of the compact space. But, as a result of the quotient by  $\Gamma$ , CHMs have no such isometries, and thus there are no massless KK gauge bosons. The non-zero KK modes once again have a mass gap that is at least as large as  $1/L$  and is more likely close to  $\sim 1/R_c$ , as in the previous cases. Thus these additional types of fluctuation should not disturb our results.

### 2.3 Radion Stabilization

In order for the CHM model to work, it is necessary to realize  $R_c \sim M_*^{-1}$  and  $e^\alpha \simeq \exp((d-1)L/R_c) \gg 1$  consistently with the ansatz of a factorizable geometry, a static internal space, and the vanishing of the 4D long-distance ( $\gg L$ ) cosmological constant (CC)  $\Lambda_4$ . To see how this might work, consider a 3-brane embedded in  $(4+d)$  dimensions, with bulk and brane actions

$$S_{\text{bulk}} = \int d^{4+d}x \sqrt{-|g_{(4+d)}|} \left( M_*^{d+2} \mathcal{R} + \Lambda - \mathcal{L}_m \right) \quad (28)$$

$$S_{\text{brane}} = \int d^4x \sqrt{-|g_{(4)}^{\text{induced}}|} \left( f^4 + \dots \right), \quad (29)$$

respectively, where  $\mathcal{L}_m$  is the bulk matter field Lagrangian, and  $f^4$  is the brane tension. Note that, since CHMs are just quotients of  $H^d$ , there will exist a uniform negative bulk cosmological constant  $\Lambda \sim M_*^{4+d}$ , and that to ensure a static internal space, this must be balanced in the field equations by the small curvature radius of the internal space. Dimensionally reducing these actions yields an effective 4D potential for the radion  $R_c$  of the form

$$V(R_c) = \Lambda R_c^d e^\alpha - M_*^4 e^\alpha (M_* R_c)^{d-2} + W(R_c) + f^4. \quad (30)$$

Here the first two terms arise from the  $(4+d)$  bulk CC term, and the curvature of the internal space. Now, in general we may expand  $W(R_c)$ , which comes from  $\mathcal{L}_m$ , as a Laurent series in  $R_c$

$$W(R_c) = \sum_p a_p \frac{M_*^4}{(R_c M_*)^p}, \quad (31)$$

with dimensionless coefficients  $a_p$ . Broadly speaking, there are then two interesting possibilities. If the determination of the minimum is dominated by a competition between any *two* terms in  $V$ , then the condition that the 4D CC vanish ( $V_{\min} = 0$ ) cannot be achieved with a brane tension such that  $|f^4| \leq M_*^4$ . Thus, such a situation is not consistent with the basic assumption that a low-energy effective theory is valid on the brane

Fortunately there is an attractive alternative. If *three* or more  $R_c$ -dependent terms in  $V(R_c)$  are all important at the minimum (for example the CC and curvature terms, and one of the matter terms from  $W$ ) then we can tune the coefficients  $a_p$  such that  $V_{\min} = 0$ , without needing  $f^4 \gg M_*^4$ . Thus, the basic assumptions remain consistent. Moreover, this tuning is particularly natural in the CHM case precisely because the minimum will naturally occur for a curvature radius close to the fundamental scale  $R_c \sim M_*^{-1}$ , at which the high-scale theory will produce many different terms that contribute roughly in an equal way. (This is exactly the opposite situation from the large flat extra dimension case where the minimum has to occur at a length scale much greater than  $M_*^{-1}$ .) This one fine-tuning is just the usual 4d CC problem. It seems unlikely to me that this one fine tuning will be solved within these models, since in the end one is left with an arbitrary higher-dimensional cosmological constant that one can add to the theory. Nevertheless, at the very least the cosmological constant problem appears in a different guise in these models.

It remains to check one important detail. In the usual large extra dimension scenario the radion moduli problem in the early universe provides quite a strong constraint<sup>17</sup>. In the CHM case this problem is much weakened.

The radion, which is the light mode corresponding to dilations of the internal space, gets its mass from the stabilizing potential  $V(R_c)$ . Here

$$m_r^2 = \frac{1}{2} \frac{R_c^2 V''(R_c)}{e^\alpha M_*^{d+2} R_c^d} \simeq \frac{1}{R_c^2}, \quad (32)$$

which is close to  $M_*^2 \sim \text{TeV}^2$ . Therefore, the radion lifetime is  $t \sim M_{\text{pl}}^2/M_*^3$ , much shorter than in the case of flat extra dimensions, and only slightly longer than the age of the universe at nucleosynthesis, even if  $M_* \sim \text{TeV}$ .

### 3 Conclusions and Further Directions

I have briefly reviewed the structure of theories with large extra dimensions, and discussed the main constraints on these models. I have then described an important modification to these theories, in which the internal manifold comprises a CHM. With this modification, the hierarchy problem is solved by a mild tuning of parameters, in an interesting and topologically stable way.

While cosmologically and astrophysically much safer, models with internal compact hyperbolic spaces do have testable signatures accessible to collider experiments. Since KK modes abound close to the fundamental scale, Standard Model particle collisions with center-of-mass energies near this scale will result in the production of KK particles, detectable by a distinctive missing energy signature<sup>6</sup>. Although this is qualitatively similar to the scenario of<sup>10</sup>, the spectrum of KK modes one will see is quite distinctive.

A full exploration of these experimental signatures will require a more complete investigation of the spectrum of large CHMs, in particular the issues of isospectrality and homophonicity of such manifolds. It is quite likely that such CHMs have other implications for higher-dimensional physics.

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